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A Landscape of Time Asymmetry

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This is a conceptual overview on a polemical subject: the problem of time asymmetry. It is proved that time asymmetry can be considered as a global generalized symmetry breaking, produced by a choice of a physically admissible state space, in a global Gel'fand triplet. The well-known physics of irreversible process can be studied using this mathematical structure and all the arrows of time can be explained and coordinated. But the deeper problems of time definition and time arrow in quantum gravity remain outside of this landscape.

1. INTRODUCTION

If we ignore the time-asymmetric weak interaction [as is usual in this kind of research (Sachs, 1987), since the weak interaction is so weak that it is difficult to see how it can explain the macroscopic time asymmetry], the time-asymmetry problem can be stated in the following question:

How can we explain the obvious time asymmetry of the universe and most of its subsystems if the fundamental laws of physics are time-symmetric?

There are only two causes for asymmetry in nature: either the laws of nature are asymmetric or the solutions of the equations of the theory are asymmetric. As time asymmetry is not an exception, the answer is contained in the question itself: If the laws of nature are time-symmetric, essentially the only way we have to explain the time asymmetry of the universe is to postulate that the space of solutions is not time-symmetric, namely to use the second cause of asymmetry.

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Let us rephrase this analysis using a mathematical language: Mathematicians say that a function is defined by two conditions:

$$f(x) = y, \qquad f: \quad D \rightarrow R$$

namely we must know not only the function itself, but also its domain of definition and its range. Physicists usually use only the first condition and consider the second one as a mathematical sophistication. The problem appears when physicists try to solve global problems, which are related to the two conditions, as local problems, i.e., using the first one only. For example, the question "is the function $f(x) = y = x^2$ is an even or an odd function?" cannot be answered. In fact this function is even only if f: $R \rightarrow R_+$, but the question has no meaning if f: $R_+ \rightarrow R_+$.

Analogously, if we consider the locally time-symmetric Liouville equation

$$i \frac{d\rho}{dt} = L\rho$$

where ρ is a distribution function or a density matrix and L the Liouville operator, and we ask ourselves if the time evolutions defined by this equation are time-symmetric or time-asymmetric, the question has no meaning. To give an answer we must define the space where the ρ lives. The time evolution will be symmetric if $\rho \in \mathcal{L}$, the usual Liouville space. The evolution will be time-asymmetric if $\rho \in \Phi_- \subset \mathcal{L}$, namely if we restrict the ρ to a subspace of asymmetric solutions. Mathematical sophistication is usually superfluous, but sometime, it is essential.

This will be the guideline or recipe of our research.

Going back to the initial question in more detail, we must also observe that if the initial state of the universe (or one of its subsystems) were an equilibrium state, the universe (or the subsystem) would remain forever in this equilibrium state, making it impossible to find any time asymmetry. Then we must solve three problems:

A. To explain why the universe (or the subsystem) began in a nonequilibrium (unstable, low-entropy) state at a time that we will call $t = 0$.

B. To define, for the period $t > 0$, some Lyapunov variable, namely a variable that never decreases (e.g., entropy) and defines an *arrow of time* (and also to find irreversible evolution equations, i.e., master equations), in spite of the fact that the main laws of physics are time-symmetric. To solve this problem we can use our recipe.

C. In different areas of physics we find different time asymmetries, namely several arrows of time (listed as they appear in the text):

- *The thermodynamic arrow of time (TAT)*: the direction of the increase of entropy.
- *The quantum arrow of time (QAT)*: the arrow defined by the collapse of the wave function, i.e., the time direction that goes from preparation to measurement (Bohm, 1995; Bohm *et al.*, n.d.; Antoniou *et al.*, 1995).
- *The global arrow of time (GAT)*: the direction of the future semicones of the oriented time manifold that we usually take as the model of our universe (Lichnerowicz, 1964).
- *The electromagnetic arrow of time (EMAT)*: the choice of retarded solutions instead of advanced solutions, which means to choose the past semicones for the propagation of the solutions and therefore essentially GAT = EMAT.
- *The psychological arrow of time (PAT)*: our feeling that past is substantially different than the future.
- *The cosmological arrow of time (CAT)*: the direction of the increasing of the radius of the universe.

Choosing a convenient cosmological model, we must demonstrate that all these arrows point to the same direction.

In the following sections we will comment and find schematic solutions (using the minimum number of mathematical equations and referring to the literature as much as possible) for these three problems. In other words, we will coordinate the solutions found by other authors and ourselves in order to present an overview on the present state of the time-asymmetry problem.

But this landscape is unfinished for two main reasons:

- We try neither to define time nor to find its real nature, as in Castagnino (1989), Castagnino and Mazzitelli (1989, 1990), Castagnino and Lombardo (1993), and Barbour (1994).
- We only use usual physics. We neither attempt to define the entropy of the gravitational field nor use quantum gravity, essentially because we think that these are quite unfinished and poorly understood area of physics that may obscure all our landscape [nevertheless see Castagnino *et al.* (1995a, 1996) and Castagnino (1996)].

But, of course, we must solve these problems to have a full comprehension of the subject.

2. THE BRANCH ARROW OF TIME

Let us begin with problem (A): The set of *irreversible* processes within the universe, each one beginning in an unstable nonequilibrium state, can be

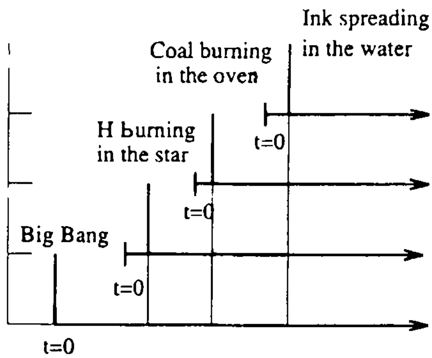

Fig. 1. The classical image of a branch system.

considered a *branch system* (Reichenbach, 1956; Davies, 1994). Namely, every one of these processes began in a nonequilibrium state such that this state was produced by a previous process of the set. For example, the Gibbs ink drop (initial unstable state) spreading in a glass of water (irreversible process) is only probable (since the probability to create an ink drop by fluctuations is extremely small) if there was first an ink factory, which extracted the necessary energy from an oven, where coal (initial unstable state) was burned (branched irreversible process); in turn, coal was created with energy coming from the sun, where H (initial unstable state) is burned (branched irreversible process); finally H was created using energy obtained from the unstable initial state of the universe (the absolute initial state of the branch system). This branch system is symbolized in fig. 1. Therefore, using this hierarchical chain, all the irreversible processes are related to the cosmological initial condition, the only one that must be explained. We will give a quantum diagram for a branch system in Section 4 and we will try to solve the problem of the initial low-entropy-unstable initial condition in Section 6. For the moment let us observe that the branch system defines its own arrow of time, the *branch arrow of time (BAT)*, as the direction that goes from the unstable initial state of every member of the system toward equilibrium. We will see that BAT is the master arrow of time that allows us to relate and coordinate all others.

3. DYNAMICS VS. THERMODYNAMICS

Problem (B) can also be considered as the search for a unified formalism of *dynamics* (namely all areas of physics with reversible equations) with *thermodynamics* (where the increasing of entropy, in irreversible processes, defines the TAT). The idea is to somehow modify the time-symmetric structure of the theory in such a way that it allow us to define Lyapunov variables (namely, variables that always increase with time).

But once we have understood the origin of the initial unstable state of each irreversible process within the universe (even if we have not yet explained the origin of the initial state of the whole universe), it is not difficult to obtain Lyapunov variables (or irreversible evolution equations), if we consider, e.g., that the subsystems where these processes take place are not isolated. If it is so, forces of a stochastic nature penetrate from the exterior of each subsystem and, as is well known, if we add stochastic terms to a time-symmetric evolution equation, we obtain time-asymmetric ones, yielding Lyapunov variables, e.g., a nondecreasing entropy (Mackey, 1989, 1992; Lasota and Mackey, 1985). We can as well consider that each subsystem has an enormous amount of information and we are able to measure, compute, and control only a part of this information, which we will call *relevant*. If we neglect the rest of the information, the *irrelevant* part, we can also obtain irreversible evolution equations and Lyapunov variables (Mackey, 1989, 1992; Lasota and Mackey, 1985; Zwanzig, 1960, 1966; Zurek, 1991; Courbage and Nicolis, 1990). We will call these methods the *coarse-graining* or usual formalism. If we would like to use more refined mathematical tools, we can substitute the stochastic perturbations by a rigging of the space of states (Bohm, 1986; Bohm *et al.*, 1989; Bohm and Gadella, 1989; Antoniou and Prigogine, 1993) obtaining what we will call the *restricted dynamics* formalism[3] [but we believe that all these formalisms are essentially physically equivalent, as we shall see (Ordoñez, 1995)]. Perhaps the last method is the best one to deal with cosmological problems, because it is specially designed for closed systems (Castagnino *et al.*, 1993a, b; Castagnino and Laura, 1994). Also, using this last method, we can see that time asymmetry is a global symmetry breaking.

Let us see how both formalisms can be derived from the same theory in the classical case: In fact, it is reasonable to think that thermodynamic laws could be demonstrated using the classical or quantum dynamical laws. But it seems that this is not the case for the second law of thermodynamics, which says that entropy increases in irreversible evolution, leading the system to a state of thermodynamic equilibrium or maximal entropy. This problem can be made precise as follows:

(i) The Liouville equation is the time-symmetric evolution equation for classical distribution functions (or quantum density matrices) ρ belonging to

[3] We shall call *restricted dynamics* the formalism developed in Bohm (1986), Bohm *et al.* (1989), Bohm and Gadella (1989), and Antoniou and Prigogine (1993), reinterpreted in the way presented in this paper.

Liouville space, $\mathscr{L}$ i.e.,

$$i \frac{d\rho}{dt} = L\rho \tag{1}$$

where L is the Liouville operator.

(ii) Any systems beginning in a unstable state reach a final equilibrium stationary state ρ_* (an observational fact). This is a time-asymmetric process that cannot be described by the Liouville equation, since a system that follows this equation never spontaneously reaches the equilibrium state.

(iii) This equation also prevents the definition of any function or functional of ρ: $F(\rho)$ (only constructed with ρ and mathematical elements of the Liouville or phase spaces) such that $\dot{F}(\rho) > 0$. Namely, it is impossible, as a consequence of the Liouville theorem, to define a Lyapunov variable, i.e., a time-growing functional of ρ, like thermodynamic entropy [e.g., the volume of the support of a characteristic distribution function ρ is time constant, Gibbs and conditional entropies are time constants (Mackey, 1989, 1992; Lasota and Mackey, 1985), etc.].

(iv) Thus entropy is the essential missing ingredient to unify dynamics with thermodynamics (theoretical problem). What we actually want is to somehow derive, from the Liouville equation, a time-asymmetric evolution that leads the system to a thermodynamic equilibrium, with a maximal entropy stationary state ρ_*.

Therefore we have an observational problem, i.e., to combine the Liouville theorem with the obvious fact that usual physical systems have a tendency to go to a thermodynamic equilibrium, and a theoretical one, to find a unified formalism for dynamics and thermodynamics. The solution of the first problem is based in a theorem by Lasota and Mackey (1985): In fact, let $\rho(x)$, $\sigma(x)$, . . . be the densities or distribution functions, functions of $x \in \Gamma$ (the phase space)[4]; let us define an inner product $(\rho|\sigma) = \int \rho^*(x)\sigma(x)$, and let $\mathscr{L}$ be the Hilbert (Liouville) space corresponding to this product, etc. As x evolves as $x(t) = S(t)x(0)$, ρ evolves as $\rho(t) = P(t)\rho(0)$ [where $P(t)$ is the Frobenius–Perron operator associated with the evolution $S(t)$]. Then;

Theorem Let $S(t)$ be an ergodic transformation with stationary equilibrium density ρ_* [for the associated Frobenius–Perron operator $P(t)$ in a phase space of finite ρ_*-measure]. Then $S(t)$ is ρ_*-mixing if and only if $P(t)\rho$ is weakly convergent to ρ_*, i.e.,

$$\lim_{t\to\pm\infty} (P(t)\rho|g) = (\rho_*|g) \tag{2}$$

for all bounded measurable functions g.

[4]The space of the ρ is the Gibbs ensemble Γ; we can use as well the Boltzmann ensemble μ (Mackey, 1989, 1992; Lasota and Mackey, 1985).

That is, if the time evolution in phase space $S(t)$, is mixing and if there is a stationary equilibrium density ρ_*, namely such that $P(t)\rho_* = \rho_*$, then (2) can be proved.

But

$$\lim_{t \to \pm\infty} P(t)\rho \neq \rho_* \tag{3}$$

Furthermore, in many cases this limit does not even exist. In fact, think again of a typical mixing system: the ink drop. It evolves maintaining its volume, but it changes its shape in such away that filaments grow from the drop and fill all the glass in a homogeneous way; this is a typical example of mixing evolution. But, as the filaments grow they become longer and thinner. In the limit $t \to \infty$ they will be infinitely long and they will have a vanishing width. This final figure is not the support of any regular, square-integrable, distribution function of $\mathcal{L}$. Therefore we have a weak limit, but we do not have a strong limit [i.e., we do not have a limit in the norm $\|\rho\| = (\rho|\rho)$ (Mackey, 1989, 1992; Lasota and Mackey, 1985)]. Precisely, the r.h.s. of (2) symbolizes just a functional over g where ρ_* is not an ordinary density.

Nevertheless we never see or measure ρ. What we see and measure are mean values of physical quantities O such that

$$\langle O \rangle_\rho = (\rho|O) \tag{4}$$

Thus what we actually see is that

$$\lim_{t \to \infty} \langle O \rangle_\rho = \langle O \rangle_{\rho*} \tag{5}$$

In fact, all the mean values of the physical quantities go to their equilibrium mean values if the evolution of the system is ρ_*-mixing. So the solution of the problem is quite easy:

(i) The Liouville theorem is embodied in (3): the system does not go (strongly) toward the equilibrium states.

(ii) The tendency toward equilibrium is embodied in (5): the mean values of all the physical quantities go to their equilibrium values.

Clearly these facts are not contradictory. We will call this solution the *nongraining* solution.

As chaotic-mixing systems are very frequent in the universe (and therefore the universe partially behaves as a mixing system), the problem is essentially solved. Observe that the limit in (2) is the same for $t \to \pm\infty$; therefore there is not a time-symmetry breaking if $\rho \in \mathcal{L}$. But, the symmetry breaking is produced if we consider that our (sub)systems belong to a branch system and therefore we cannot consider the time $t < 0$.

What is left to be studied is item (iv), namely the different techniques to deal with the detailed calculations using a unified dynamical–thermodynamic formalism. These techniques try to find some logical modification of the theory in order to find the missing limit (3), which, even if unnecessary from the mathematical point of view, is the way physicists used to think (or loved to think), at least up to now. Also, the strong limit is necessary in order to find, in an easy way, the right thermodynamic behavior for some definitions of entropy (Mackey, 1989, 1992; Lasota and Mackey, 1985). In fact there are two techniques, as follows:

3.1. Coarse-Graining

Let us define an arbitrary, but time-independent, projector

$$P = |g)(g|, \ldots, (g|g) = 1 \tag{6}$$

and let us define a coarse-graining density function as

$$\tilde{\rho} = P\rho = |g)(g|\rho) \tag{7}$$

Since complex physical systems usually have a great number of dynamical variables (these variables define the microscopic state of the system), and we just measure, see, or control a small number of them (the ones that define the macroscopic state of the system), we will say that the later information is *relevant*, while the former is *irrelevant*, so it can be neglected. We consider that all the relevant information can be obtained by a projection and is precisely contained in $\tilde{\rho}$. From (2) we have

$$\lim_{t\to\infty} |g)(g|P(t)\rho) = |g)(g|\rho_*) \tag{8}$$

and therefore we have the strong limit

$$\lim_{t\to\infty} \tilde{\rho}(t) = \tilde{\rho}_* \tag{9}$$

which would be the coarse-graining version of (3) and the main equation of the first technique [of course the same thing happens with the general projector $\Pi = \Sigma|g_i)(g_i|$, $(g_i|g_j) = \delta_{ij}$]. It is easy to demonstrate that (9) is a limit in norm. From the above equations Lyapunov variables and entropy can be defined and master equations can be found [if some extra conditions are fulfilled (Courbage and Nicolis, 1990); these conditions are weaker as the systems become more chaotic].

But it is also evident that (9) can be obtained with a quite arbitrary state $|g)$ and that all the philosophy typical of the coarse-graining technique namely the definition and consideration of macroscopic and microscopic states (Zwanzig, 1960, 1966; Zurek, 1991) is just an intuitive justification to give a physical meaning to the limit (9). But as this justification is really unnecessary, since the relevant and important limit is (2), the physical explanation is redundant and all the philosophy of the coarse-graining technique can therefore be philosophically criticized (Prigogine, 1980). This is the main problem with coarse-graining. It is an arbitrary method. It works perfectly well, but it is difficult to justify, based on physical-philosophical (metaphysical) arguments.

In fact, coarse-graining contains the misleading statement: *we cannot see microscopic states* (i.e., ρ), *but we can see macroscopic states* (i.e., $\tilde{\rho}$). This statement leads to the problem of finding a unique and reasonable definition for these macrostates. This problem is unsolved and, in our opinion, it will remain unsolved, since $|g)$ is essentially arbitrary. Also, if we arbitrarily choose some definition of macrostates, we are introducing a physical element that really is alien to the system itself, and therefore this definition, even if natural in particular examples, will be suspicious from a general point of view.

The correct "no-graining" statement [at least to solve problem (ii)] is: *we cannot directly measure microscopic states* (i.e., ρ), *we can only measure mean values of physical quantities or observables* [among them the projector $P = |g)(g|$ and therefore the arbitrarily defined macroscopic states]. This statement is completely true at the classical (and also the quantum) level (Castagnino and Laura, 1997a) and refers to *all physical observables if the system is mixing*. Then we can rigorously say, e.g., that the two thermodynamic average variables $\langle p\rangle$ and $\langle v\rangle$ (i.e., the average pressure and specific volume) define the thermodynamic macrostate of a perfect gas, etc.

3.2. Restricted Dynamics

As we have shown, the main achievement of coarse-graining is the strong limit (9). Let us see how we can obtain a similar result using another mathematical structure, and in this way obtain the restricted dynamics formalism. Since the weak limit (2) is really a functional limit, this formalism essentially consists in using the mathematical of functionals. In fact, let $\mathcal{L}$ be the usual Hilbert–Liouville space of the state function ρ, and $\mathcal{L}^{\times} = \mathcal{L}$ the space of the linear operator on $\mathcal{L}$. We may think that not all $\rho \in \mathcal{L} = \mathcal{L}^{\times}$ are physically admissible states. In fact:

(i) Practically, real physical state functions are only measured at a finite number of points of phase space Γ and then they are interpolated as, e.g., polynomials (that belong to a space that we will call $\mathcal{P}$) which do not have sophisticated mathematical behaviors, e.g., they are continuous and derivable functions and not discontinuous, nonderivable functions, even if square-integrable. So it is reasonable that $\rho(x)$ would be at least a Schwarz function, namely a continuous, infinitely derivable function, well-behaved in the eventual infinities of phase space (we will make this definition precise in the next section). So let us call Φ_- the space of physically admissible states, (for the moment) an arbitrary space such that it is a complete space in some topology stronger than the topology of $\mathcal{L}$, dense in $\mathcal{L}_-$ a subspace of $\mathcal{L}$, and such that

$$\Phi_- \subset \mathcal{L}_- = \mathcal{L}_-^\times \tag{10}$$

Also, $\mathcal{P}$ will be dense in Φ_-, in such a way that if we complete $\mathcal{P}$ with the topology of Φ_- we will obtain this space. If we consider the dual $\Phi_-^\times$ of Φ_-, we have a Gel'fand triplet:

$$\Phi_- \subset \mathcal{L}_- = \mathcal{L}_-^\times \subset \Phi_-^\times \tag{11}$$

The topology of $\Phi_-^\times$ will be weaker than that of $\mathcal{L}_-$.

(ii) From Lasota–Mackey theorem we know that the function g of (2) and therefore the function O of (4) are just bounded, measurable functions and therefore they belong to a space larger than $\mathcal{L}_-$, say a space $B \supset \mathcal{L}_-$. So we will postulate that:

(a) Every physically admissible state ρ belongs to the space Φ_- such that $\Phi_- \subset \mathcal{L}_-$.

(b) Every observable belongs to space B, such that $\mathcal{L}_- \subset B \subset \Phi_-^\times$.

(iii) Furthermore, we know that not all the evolutions of state functions are physically admissible. The physically admissible evolutions are those that appear in the branch system, namely those that begin in an unstable state and go toward equilibrium (Gibbs ink drop spreading in a glass of water, a sugar lump dissolving in a cup of coffee, etc.). The physically inadmissible evolutions can be obtained by the time inversion of the admissible ones; therefore they begin in an equilibrium state and evolve toward an unstable state (the ink or the sugar concentrating spontaneously and creating a drop or a lump). This kind of evolution does not appear in the branch system nor in nature, because the spontaneous appearance of an unstable state by a fluctuation is highly improbable. Practically, unstable states are built before the instant of creation of the subsystem considered (the instant when we put the ink in the water or the lump in the coffee) by the action of a larger subsystem (the ink or the sugar factories) which also belongs to the branch system, but contains other processes that generate energy and go toward equilibrium (coal burning in the factory's oven). So Φ_- will be the space of these admissible states, namely the space of the introduction, where we will use our main recipe. If T is the time inversion operator, the space of inadmissible states is Φ_+, T: $\Phi_- \to \Phi_+$, and we also know that $\Phi_- \neq \Phi_+$. Furthermore, all the evolution of any admissible state takes place within the apace Φ_-. Thus, using the usual terminology of axiomatic field theory (Bogulubov *et al.*, 1975), we will say that Φ_- will be the space of regular states, $\mathcal{L}_-$ the space of usual states, and $\Phi_-^\times$ the space of generalized states. In the limit of

$t \to \infty$ some states may belong to $\Phi_-^\times$; e.g., from (2) we see that ρ_* normally belongs to this space (really to space $B \subset \Phi_-^\times$).

(iv) Now usually we choose Φ_- to be a nuclear space with a nuclear topology, stronger than the norm topology of Liouville space $\mathcal{L}$, and then $\Phi_-^\times$ will be another nuclear space, with a nuclear topology weaker than the norm topology of $\mathcal{L}_-$.

Once we have chosen the space Φ_- endowed with all these properties we can say that, as ρ and ρ_* can be considered as functional on $B \subset \Phi_-^\times$, from (2) it can be proved that

$$\lim_{t\to\infty} P(t)\rho = \rho_* \tag{12}$$

and we have found a different way to obtain a "strong" limit (precisely, a functional or distribution limit, with the convergence of the nuclear topology of $\Phi_-^\times$) corresponding to (2). So the new recipe consists in going from the realm of Hilbert space to the realm of functionals or mathematical distributions, which are endowed with a weaker topology, in such a way that the weak limit of the Lasota–Mackey theorem becomes a limit in this weaker topology. So both methods reach the same goal, to go from a weak limit to a "strong" one, using projection in the case of coarse-graining, inventing a new topology in the case of restricted dynamics.

The problem is that there is no general method to define Φ_-. Nevertheless, as we will see, we have more general methods in the quantum case to define Φ_-, a fact that supports the hope that also the classical problems could be solved. But in specific examples (Antoniou and Tasaki, 1991, 1993; Castagnino *et al.*, 1997b) a space Φ_- is defined in such a way that Lyapunov variables, an entropy, and master equations can be defined as in the coarse-graining case. These examples show that if spaces Φ_- and Φ_+ are properly defined, it can be proved that:

(i) The space $\Phi_+^\times(\Phi_-^\times)$ contains pure decaying (growing) states, having a damping factor $e^{-\gamma t}$ (a growing factor $e^{\gamma t}$) in their evolution ($\gamma \geq 0$). These are the spaces of the generalized states (similar to planes waves or Dirac deltas) that appear in useful spectral decompositions.

In fact, a complete set of right eigenvalues of the Liouville operator $\{|n-)\}$ such that

$$L|n-) = z_n|n-) \tag{13}$$

where $z_n = \omega_n - i\gamma_n$, $\gamma_n \geq 0$, and $|n-) \in \Phi_+^\times$ exists. A complete set of left eigenstates $\{|n+)\}$ such that

$$(n+|L = z_n^*(n+| \tag{14}$$

where $|n+) \in \Phi_-^\times$ also exists. If $|\varphi_-) \in \Phi_-$ and $|\sigma_+) \in \Phi_+$, then

$$(\varphi_+|\varphi_-) = \sum_n (\varphi_+|n-)(n+|\varphi_-) \tag{15}$$

in such a way that we can expand in a weak sense any $|\varphi_-) \in \Phi_-$ as

$$|\varphi_-) = \sum_n |n-)(n+|\varphi_-) \tag{16}$$

$|n-)$ evolves as

$$|n(t)-) = e^{-iz_n^* t}|n-) = e^{-i\omega_n t}e^{-\gamma_n t}|n-) \tag{17}$$

and as $\gamma_n \geq 0$ it is a decaying generalized state. Symmetrically, $|n+)$ is a growing generalized state.

(ii) The usual evolution group, with evolution operator e^{-iLt} (where L is the Liouville operator) valid for $-\infty < t < \infty$, is split into two semigroups, one acting on Φ_-, with evolution evolution operator $U_-(t)$, namely $U_-(t)$: $\Phi_- \to \Phi_-$ valid for $t > 0$, with an evolution operator that cannot be inverted, the other acting in Φ_+, with evolution operator $U_+(t)$, namely $U_+(t)$: $\Phi_+ \to \Phi_+$, valid for $t < 0$, also with noninvertible evolution operator. So if we restrict the dynamics to the space of admissible states Φ_-, we cannot invert the evolution operators $U_-(t)$. This fact shows that we have obtained an *irreversible evolution* so we have really found a time-asymmetric theory. In other words, if we take Φ_- as the state of physical states, we have really broken the time symmetry, since we cannot find an inverted time operator $U_-^{-1}(t) = U_+(t)$ for $t > 0$, acting within Φ_-.

(iii) A state $\rho \in \Phi_-$ can be expanded in eigenvectors of the Liouville operator, with complex eigenvalues, that belong to $\Phi_+^\times$; then the time evolution or ρ is an expansion of decreasing exponentials,

$$\rho(t) = \rho_* + e^{-\gamma t}\rho_1 + \cdots \tag{18}$$

where $\rho_* = \text{const} \cdot e^{-\gamma t}$ is the slowest decreasing exponent, and the dots symbolize higher powers of this exponent or faster decreasing exponents.

The normalization conditions at any time t yields

$$\text{tr}\,\rho(t) = \text{tr}\,\rho_*(t) = 1, \ldots \text{tr}\,\rho_1 = 0 \tag{19}$$

The last equations show that ρ_1 is not a state, but only the coefficient of a correction around the equilibrium state ρ_*. It is explicitly proved in Castagnino and Laura (1997a) that ρ_1 has a vanishing trace.

(iv) The conditional entropy

$$S(\rho|\rho_*) = -\int_\Gamma \rho(x) \log \frac{\rho(x)}{\rho_*(x)}\, dx \tag{20}$$

never decreases. In fact, it can be proved that if $P(t)$ is a Markov operator and ρ and σ are two densities, then (Mackey, 1989, 1992; Lasota and Mackey, 1985)

$$S(P(t)\rho|P(t)\sigma) \geq S(\rho|\sigma) \tag{21}$$

If the evolution is reversible, we must have the equal sign, since t can be either positive or negative. Now as $U_-(t)$ is an irreversible Markov operator and ρ_* is a stationary density, i.e., $U_-(t)\rho_* = \rho_*$, then if $\rho \in \Phi_-$:

(a) Operator $U_-(t)$ is well defined only if $t > 0$; therefore it is possible that

$$S(U_-(t)\rho|\rho_*) > S(\rho|\rho_*) \tag{22}$$

(b) Anyhow S is necessarily nondecreasing, e.g., it cannot have an oscillatory behavior.

(c) But we can compute the conditional entropy (20) using (18) and (19) and we obtain[5]

$$S(\rho(t)|\rho_*) = -\int_\Gamma \rho \log \frac{\rho}{\rho_*} dx = -e^{-\gamma t} \int_\Gamma \frac{\rho_1(x)^2}{\rho_*(x)} dx + \cdots \tag{23}$$

where the dots symbolize faster decreasing exponents.

So, now we are sure that the conditional entropy always grows.

Perhaps the main surprise with the restricted dynamics technique is that we now work in the space Φ_-, where no "unphysical" states have been added since $\Phi_- \subset \mathcal{L}$. Also, the space Φ_- is dense in the space $\mathcal{L}_-$, so if someone would say that $\mathcal{L}_-$ really is the space of "physical states," these states can be approximated by regular states of the space Φ_- as close as we want.

But the space Φ_- (the mathematical object that defines the restricted dynamics) is arbitrary up to a certain point. It fact, Φ_- is simply a space of undoubtedly physical states $\mathcal{P}$ completed with a topology in such a way that T: $\Phi_- \neq \Phi_-$. We do not have a general method to define this topology in an arbitrary case. But we believe that it will be easier to find a canonical physically unique time-asymmetric topology than a coarse-graining. [In fact, in Bohm (1995) and Bohm *et al.* (1995b) it is claimed that there is only one way to choose Φ_- in the quantum case.]

[5] As in this equation distributions are multiplied, some care must be taken in order to convince ourselves that what we are doing is mathematically correct. For example, the distributions can be transformed in ordinary density matrices by a Λ transformation (Castagnino *et al.*, 1997c). Λ is a transformation such that $\Lambda^{-1}|n+\rangle \in \mathcal{H}$; then we can define $\rho_\Lambda = \Lambda^{-1}\rho$, $S = S(\rho_\Lambda|\rho_{\Lambda *})$ (the exponent "-1" is just a way to keep the traditional notation). This transformation maintains the damping factors, so the results obtained remain valid, but the distributions become ordinary matrices, which can be multiplied. It is precisely in this way that the r.h.s. of (23) becomes a well-defined expansion of decreasing exponentials. There are also more refined mathematical ways to reach to the desired result, such as that of (Castagnino and Laura, 1997a), Part III, and those based on the theory of locally convex spaces (in preparation).

So neither of the two formalisms is completely sinless. Nevertheless, as the real physical problem is solved by the Mackey and Lasota theorem, we can say that all these sins are venial sins. On the other hand, both approaches have some advantages, e.g.:

(i) Coarse graining works just with one physical space, $\mathcal{L}$. Also, coarse-graining averages are unavoidable to calculate global thermodynamical variables like temperature or pressure, but:

(ii) The time evolution of $\rho(t)$ can be computed more easily using the restricted dynamics approach since we have the vectors of space $\Phi_+^\times$, that can be used to find new spectral expansions for the observables and the states of the problem. Once we know $\rho(t)$, we can compute average states like $\tilde{\rho} = P\rho(t)$, while the direct computation of $\tilde{\rho}(t)$ using coarse-graining technique directly can be more difficult (Zwanzig, 1960, 1966; Zurek, 1991).

At this point we can see that coarse-graining and restricted dynamics are both based on the idea that we never have complete information on the system we are working with. In coarse-graining we have a "coarse" lack of information, because we neglect the irrelevant part. In restricted dynamics we have a "fine" lack of information—we ignore the topology and we are forced to choose one. So we can complete our state space $\mathcal{P}$ with some time-asymmetric topology and get Φ_- also obtaining a time-asymmetric theory.

4. THE STATUS OF QUANTUM RESTRICTED DYNAMICS

In this section we will briefly rephrase what we have said, but in the quantum case, and we will see how the Gel'fand triplet structure naturally appears. The diagrams (Bohm diagrams) that we will obtain will be our best conceptual tool to understand the relations among the different arrows of time, because they graphically represent complicated calculations.

In fact, as several quantum observables are not even well defined in Hilbert space $\mathcal{H}$, rigorous quantum mechanics is formulated in a Gel'fand triplet (Bogolubov *et al.*, 1975):

$$\mathcal{S} \subset \mathcal{H} \subset \mathcal{S}^\times \tag{24}$$

where:

$\mathcal{S}$ is the space of "regular states"or test function space corresponding to Schwarz-class wavefunctions, which we will consider the real physical states.

$\mathcal{H}$ is the space of "states," or Hilbert space, introduced to extend the notion of probability to a larger space. These states are square-integrable wave functions, e.g., Schwarz functions where a finite set of points is removed from the surface representing the function. Therefore they are not derivable and so it is impossible to compute, e.g., the momenta or the energy using these kinds of functions.

$\mathcal{S}^{\times}$ is the space of "generalized states," or rigged Hilbert space, namely the space of functionals over $\mathcal{S}$ (like plane waves or Dirac deltas) that are essentially used to find the spectral expansion of the regular states.

Let K be the Wigner or time-inversion operator. In the usual time-symmetric or reversible quantum mechanics the evolution Hamiltonian H is time-symmetric, i.e.,

$$KHK^{\dagger} = H \tag{25}$$

In fact, if it were time-asymmetric, the theory would be trivially time-asymmetric, and we know that such a trivial theory does not coincide with physical reality. In the wave function representation K coincides with complex conjugation, so it is defined over $\mathcal{S}$ by

$$K\varphi(x) = \varphi^{*}(x) \tag{26}$$

Therefore

$$K\colon \quad \mathcal{S} \to \mathcal{S} \tag{27}$$

But the real universe and macroscopic objects clearly have time-asymmetric evolutions, so we must explain how this time asymmetry appears if the basic quantum mechanical laws of the universe are time-symmetric. Also, at this quantum level, the usual and successful explanation is based on coarse-graining: macroscopic objects have a huge number of dynamical variables and we can only measure and control a small number of them, the so-called relevant variables. If we neglect the rest of the variables, the irrelevant ones, we obtain time-asymmetric evolution equations (Zwanzig, 1960, 1966; Zurek, 1991).

Nevertheless, in this section [according to the line of thought pioneered in Bohm, (1986, 1995), Bohm *et al.* (1989, 1995b), Bohm and Gadella (1989), Antoniou and Prigogine (1993), and Sudarshan *et al.* (1978)] we want to stress the importance of restricted dynamics, because we believe that the development of an alternative theory will enhance our knowledge about time-asymmetry and give us new quantum mechanical tools (e.g., Bohm diagrams). Thus we want to sketch an irreversible quantum theory which explains time asymmetry directly from the basic microscopic level. In this way we will have two (probably equivalent) theories to compare.

Obviously we want to obtain our new theory making minimal changes to the well-established and usual quantum mechanics. If we change (25) or (26), we are almost sure to find experimental problems. So the minimal modification is to change (27), defining a new test function space $\phi_{-} \subset \mathcal{S}$ such that

$$K\colon \quad \phi_{-} \to \phi_{+} \neq \phi_{-} \tag{28}$$

In this way K is not even defined over the space of regular states ϕ_- and naturally time asymmetry appears. From now on ϕ_- will take the role of $\mathcal{S}$, so regular states will belong to ϕ_-, etc.

It can be demonstrated that an irreversible quantum theory based on a Gel'fand triplet

$$\phi_- \subset \mathcal{H}_- \subset \phi_-^{\times} \tag{29}$$

is feasible and yields reasonable physical results, such as the decaying of unstable states, decoherence, the golden rule, etc., if test function space ϕ_- is properly chosen. We can also show that what it is done in papers cited above is essentially our minimal modification of the ordinary reversible quantum theory (Castagnino and Laura, 1997a). But with this new approach we gain a clearer comprehension of the extension from the reversible quantum theory to the irreversible one described in these papers.

Let us consider a quantum system with a spectrum endowed with a continuous component ω, say $0 \leq \omega < \infty$ (the presence of this continuous spectrum seems necessary to construct the theory also in the classical case, since the mixing evolution operators have continuous spectra). Then the test function space $\mathcal{H}_-$ is chosen such that, if $\varphi \in \mathcal{H}_-$, then in the energy representation $\varphi(\omega) \in H^2_-$, i.e., Hardy class from below [essentially the analytical continuation of $\varphi(\omega)$ is holomorphic in the lower half-plane]. If also $\varphi(\omega) \in \mathcal{S}$ then $\varphi \in \phi_- \subset \mathcal{H}_-$. Then, in usual scattering models with resonances (Bohm, 1986, 1995; Bohm *et al.*, 1989, n.d.; Bohm and Gadella, 1989; Antoniou and Prigogine, 1993) it is shown that:

(i) Condition (28) is fulfilled and ϕ_+ contains all functions such that $\varphi(\omega) \in H^2_+$, the Hardy class from above. Also, $\varphi(\omega) \in \mathcal{S}$.

(ii) $\phi_+^{\times}(\phi_-^{\times})$ contains pure decaying (growing) states, having a damping factor $e^{-\gamma t}$ (a growing factor $e^{\gamma t}$) in their evolution ($\gamma \geq 0$), as we will see.

Each lower (upper) half-plane simple pole of the S-matrix z_n, Im $z_n <$ 0 (Im $z > 0$), corresponds to a left-eigenstate of the (perturbed) Hamiltonian $|n-\rangle \in \phi_+^{\times}(|n+\rangle \in \phi_-^{\times})$ with eigenvalue z_n which is a decaying (growing) Gamow vector. Ordinary states can be expanded on bases containing these Gamow vectors: e.g., any state of ϕ_- can be expanded in eigenvectors of the Hamiltonian belonging to the space $\phi_+^{\times}$. These expansions are useful to compute time evolutions, since these bases are adapted to the evolution of unstable states (not using these bases as a computation device is as inconvenient as not using spherical waves in scattering problem with spherical symmetry).

In fact, a complete set of right eigenvalues of the Hamiltonian operator $\{|n-\rangle\}$ exists such that

$$H|n-\rangle = z_n|n-\rangle \tag{30}$$

where $z_n = \omega_n - i\gamma_n$, $\gamma_n \geq 0$, and $|n-\rangle \in \phi_+^\times$. A complete set of left eigenstates $\{|n+\rangle\}$ such that

$$\langle n+|H = z_n^*\langle n+| \tag{31}$$

where $|n+\rangle \in \phi_-^\times$, also exists. If $|\varphi_-\rangle \in \phi_-$ and $|\varphi_+\rangle \in \phi_+$, then

$$\langle\varphi_+|\varphi_-\rangle = \sum_n \langle\varphi_+|n-\rangle\langle n+|\varphi_-\rangle \tag{32}$$

in such a way that we can expand in a weak sense any $|\varphi_-\rangle \in \phi_-$ as

$$|\varphi_-\rangle = \sum_n |n-\rangle\langle n+|\varphi_-\rangle \tag{33}$$

$|n-\rangle$ evolves as

$$|n(t)-\rangle = e^{-iz_nt}|n-\rangle = e^{-i\omega_nt}e^{-\gamma_nt}|n-\rangle \tag{34}$$

and as $\gamma_n \geq 0$, it is a decaying generalized state. Symmetrically, $|n-\rangle$ is a growing generalized state.

Usually the sum in (32) and (33) represents a sum and an integral;

$$\langle\varphi_+|\varphi_-\rangle = \sum_n \langle\varphi_+|n-\rangle\langle n + |\varphi_-\rangle + \int_0^\infty \langle\varphi_+|\omega-\rangle\langle\omega+|\varphi_-\rangle \, d\omega \tag{35}$$

where each index n corresponds to a pole z_n of the S-matrix. The continuous spectrum $0 \leq \omega < \infty$ is usually real, although in some cases complex continuous spectra studied (Sudarshan, 1993; Gadella and Rudin, 1996).

(iii) The usual evolution group, with evolution operator e^{-iHt}, valid for $-\infty < t < \infty$, is split into two semigroups, one acting on ϕ_-, with evolution operator $U_-(t)$ valid for $t > 0$, such that it cannot be inverted, the other acting in ϕ_+, with evolution operator $U_+(t)$, valid for $t < 0$, also a noninvertible evolution operator. The fact that we cannot invert these evolution operators shows that we have really found a time-asymmetric theory.

(iv) An "in" stable eigenstate $|n\rangle \in \mathcal{H}$ of the unpreturbed Hamiltonian (defined in the far past) is transformed by the interaction into a growing eigenstate $|n+\rangle \in \phi_-^\times$ of the total Hamiltonian, representing the unstable state ion its creation process, and a damped eigenstate $|n-\rangle \in \phi_+^\times$ of the same perturbed Hamiltonian, representing the corresponding unstable state in the decaying process (which goes to an "out" stable state $|n\rangle \in \mathcal{H}$, in the far future).

(v) Mixed decaying states can be studied in the Gel'fand triplet

$$\Phi_- \subset \mathcal{L} \subset \Phi_-^\times$$

where $\Phi_- = \phi_- \otimes \phi_-$, the quantum Liouville space is $\mathcal{L} = \mathcal{H} \otimes \mathcal{H}$, etc. The same holds for growing states, changing $-$ by $+$. These quantum spaces are related via the Wigner integral to the corresponding classical ones (Castagnino *et al.*, 1997). Using quantum (or classical) mixed states, it can be proved that the time evolution of a state of $\rho(t) \in \Phi_{-}^{\times}$ reads [cf. (18)]

$$\rho(t) = \rho_{*} + e^{-\gamma t}\rho_{1} + \cdots \tag{36}$$

where the dots symbolize higher powers of the exponent $e^{-\gamma t}$ or faster decreasing terms. Using this evolution, we can find a nondecreasing conditional entropy [cf. (23)]

$$S = -\mathrm{tr}[\rho \log(\rho_{*}^{-1}\rho)] \tag{37}$$

In order to understand the relation among these states, we can compute the survival probability of state $|n\rangle$: $P(t) = |\langle n|n(t)\rangle|^{2}$. This probability is shown in Fig. 2, where we can see that there is a period, around time $t = 0$, that corresponds to the transition from the creation process to the decaying process (Zeno period), two large periods with exponential behavior (one growing, the other decaying), and two (initial and final) period (Khalfin periods). The figure is symmetric, because $|n\rangle \in \mathcal{H}$, and this space has no time asymmetry. The survival probabilities of unstable states $|n-\rangle \in \phi_{+}^{\times}$ and $|n+\rangle \in \phi_{-}^{\times}$ are shown in Fig. 3. They have only a unique exponential behavior period, showing the nature of these Gamow vectors; they either grow or decay eternally. They are as eternal as plane waves that belong to $\mathcal{S}^{\times}$. Therefore these Gamow vectors represent the exponential periods of the time evolution of $|n\rangle$, where the Zeno and Khalfin effects are eliminated. Also each curve in Fig. 3 is asymmetric to the other one, showing that time symmetry appears in spaces $\phi_{+}^{\times}$ and $\phi_{-}^{\times}$.

Let $\varphi \in \mathcal{H}_{-}$; as ϕ_{-} is dense in $\mathcal{H}_{-}$, the state φ can be approximated as closely as we want by a sequence of states $\varphi_{-}^{(n)} \in \phi_{-}$, and also another sequence of states $\varphi_{+}^{(n)} \in \phi_{+}$ for $\varphi \in \mathcal{H}_{+}$. Then, if $\varphi = \varphi(0)$ is considered a state at a time $t = 0$, we have that:

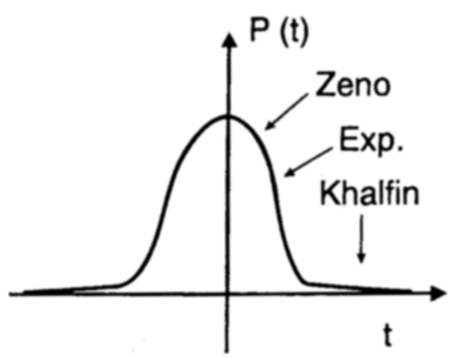

Fig. 2. The survival probability of a state $|n\rangle$.

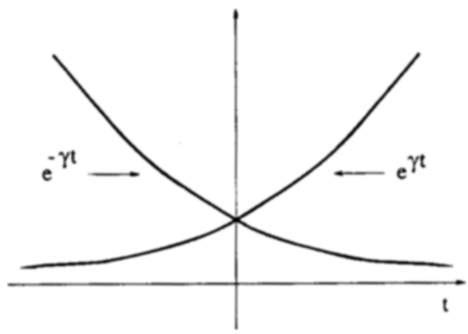

Fig. 3. The survival probabilities of states $|n+\rangle$ and $|n-\rangle$.

- The evolution of the creation process is $\varphi_+(t) = U_+(t)\varphi_+(0)$ for $t < 0$.
- The evolution of the decaying process is $\varphi_-(t) = U_-(t)\varphi_-(0)$ for $t > 0$.

To have a graphic idea of the nature of the unstable states, let us draw the ordinary diagram of a scattering (Fig. 4). In the center of the diagram there is a black box that symbolizes any resonant scattering process. A set of stable "in" states $a_1, a_2, \ldots$ is transformed by the scattering process into another set of stable "out" states $b_1, b_2, \ldots$. It is a reversible process because the evolution equations are time-reversible, so we can interchange the "in" and "out" states and all the results remain valid. In fact, Fig. 4 is essentially symmetric, like the curve in Fig. 2.

Now, let us cut the black box into two parts by a dotted line drawn at $t = 0$. Then, we can consider the right side of the figure, namely Fig. 5. This figure was introduced by Bohm (1986), so we will call this kind of figure a Bohm diagram. In Fig. 5 the set of stable "in" states creates a set of unstable states $u_1, u_2, \ldots$ which are growing states and they belong to space $\phi_-^\times$. (e.g., radiation exciting an electron of the ground state). As the states of ϕ_+ are linear combinations of the states of $\phi_-^\times$, in some sense they can also be considered as growing states and they can be symbolized as horizontal lines

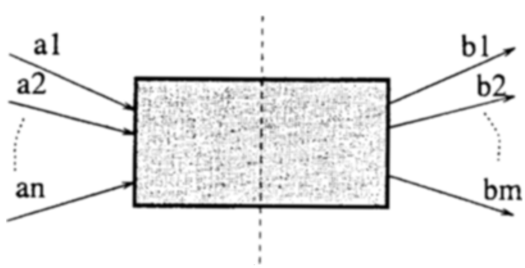

Fig. 4. Ordinary scattering diagram, with a black box.

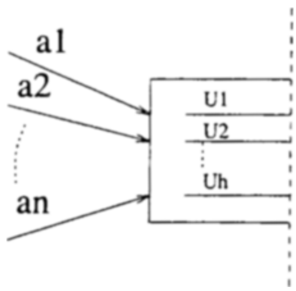

Fig. 5. Bohm diagram for growing states.

inside the half box. Figure 5 is asymmetric (like the growing hyperbola of Fig. 3) and it symbolizes an irreversible creation process. The evolution equations are still time-symmetric, but irreversibility is introduced by the growing nature of the states of the space $\phi_-^\times$ or by the noninvertible semigroup acting on ϕ_+ for $t < 0$ (namely by a generalized symmetry breaking). Using this space, we introduce an arrow of time, precisely the QAT.

We can also consider the second half of Fig. 4, namely Fig. 6. It is the Bohm diagram of a decaying process where a set of unstable decaying states $u_1, u_2, \ldots$ that belongs to $\phi_+^\times$ is transformed into a set of stable "out" states (an excited electron decaying into the ground state and emitting radiation). Figure 6 is asymmetric (like the decaying hyperbola of Fig. 3) and symbolizes a decaying irreversible process. Again, the evolution equations are still time-symmetric, but the decaying nature of the states of the space $\phi_+^\times$ introduces the irreversibility (by generalized symmetry breaking), etc. If we use this space, we can also introduce a QAT.

Bohm diagrams allow us to see the quantum structure of a branch system (Fig. 7). The universe is represented by a set of scattering processes with one initial unstable state symbolized by the cut box (at "big-bang" time $t = 0$) on the far left. Each subsystem going from an unstable state to equilibrium (e.g., the ink drop spreading in water) is symbolized by a decaying

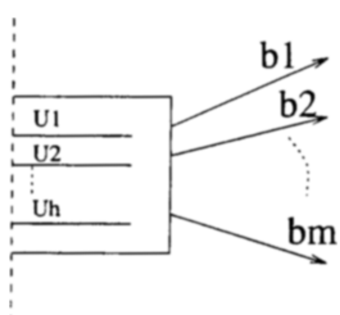

Fig. 6. Bohm diagram for decaying states.

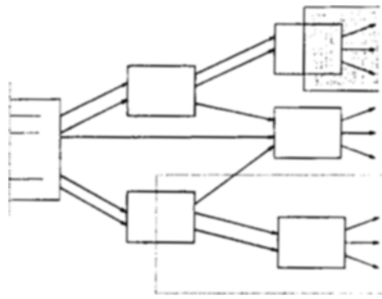

Fig. 7. Bohm diagram for the branch system.

process like the one of Fig. 6, namely the diagram in the shaded box of Fig. 7. The creation of an unstable state is symbolized by a creation process (like the one of Fig. 5) where energy comes from a previous decaying process (e.g., the ink factory with its oven). One of these larger subsystems is represented in the dotted box in Fig. 7. The overall process is irreversible, because Fig. 7 is asymmetric, and if we were to make a model of this universe (Castagnino *et al.*, 1996b; Castagnino and Laura, 1994) the state of the universe must belong to some global space $\phi_{+}^{\times}$ or $\Phi_{+}^{\times}$. Therefore in this diagram there is a clear arrow of time. But in the previous diagrams (Fig. 5 or Fig. 6) the arrow of time a was "local" one, while in this diagram it has one of the most important characteristics of the observed time asymmetry: it is global. This is the way to introduce the arrow of time in the restricted dynamics formalism: by a global and generalized (since there is not a potential field with two symmetric minima) symmetry-breaking process.

To continue with our discussion it is imperative to introduce some terminology.

5. CONVENTIONAL VS. SUBSTANTIAL

Someone might say that we have introduced the arrow of time "by hand" when we chose the space ϕ_- or ϕ_+ as the space of physical states. In order to answer this criticism, we must define two important words: "conventional" and "substantial."

- In mathematics we are used to working with identical objects, like points, the two directions of an axis, the two semicones of a light cone, the two time orientations of a time-oriented manifold, etc.
- In physics there are also identical objects, like identical particles, the two spin directions, the two minima of a typical "two-minimum" potential, etc.
- When (Sachs, 1987; Penrose, 1979) we are forced to call two identical objects by different names we will say that we are establishing a

conventional difference, e.g., when we call e_1 and e_2 two electrons, or "up" and "down" two spin directions, or "right" and "left" two minima of a symmetric potential curve; while;
- If we call two different objects by different names, we will say that we are establishing a *substantial difference*.

The problem of time asymmetry is that, in all normal time-symmetric physical theories, usually the difference between past and future is just conventional. In fact, we can change the word "past" by the word "future" in these theories and nothing changes ("in" states are only conventionally different from "out" states in Fig. 4 and they can be interchanged). But we have the clear psychological feeling that the past is substantially different than the future. Thus the problem of the arrow of time is to find theories where past is substantially different than future, such that the usual well-established physics remains valid. Our minimal irreversible quantum mechanics of Section 4 is one of these theories.

In fact, the difference between the global ϕ_- and the global ϕ_+ of the whole universe branch system is just conventional, since these two spaces are identical (as identical as the two minima of a potential). Thus physics is the same in ϕ_- as in ϕ_+. In a cosmological model (Fig. 7) life will be the same, in this universe with a quantum state in space ϕ_-, as in the universe of Fig. 8, the time-inverted image of Fig. 7, with quantum state in space ϕ_+. In fact, since in both models of the universe (if completely computed) all the arrows of time must point to the same direction, there is no physical way to decide if we are in one model or the other. So both models are identical. Thus the choice between ϕ_- and ϕ_+ is just conventional and physically irrelevant (like the choice of one of the two minima of the potential in spontaneous symmetry breaking).

But once this choice is made a substantial difference is established in the model, e.g., the only time evolution operator is $U_-(t) = e^{-iHt}$, $t > 0$, and it cannot be inverted; we have equilibrium only toward the future, etc. (as when we choose one of the two minima of the potential, a substantial time asymmetry appears in a spontaneous symmetry breaking).

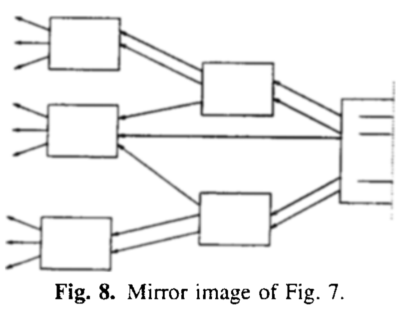

Fig. 8. Mirror image of Fig. 7.

Once the ϕ_- or the ϕ_+ is chosen in the global branch system we are forced to choose the corresponding spaces in the local subsystems if we want to study these subsystems as isolated systems, and a global arrow of time is established.

Thus, the choice between ϕ_- and ϕ_+ is trivial and unimportant, which is why the arrow of time is not introduced by hand in the restricted dynamics theory. The important choice is between $\mathcal{H}$ (or $\mathcal{S}$) and ϕ_- (or ϕ_+) as the space of our physical states. And we are free to make this choice, since a good physical theory begins by the choice of the best mathematical structure to mimic nature. *Thus, our thesis is essentially that time-asymmetric mathematical structures mimic better the time-asymmetric nature where we live than do time-symmetric mathematical structures.*

6. THE ENTROPY GAP

Let us go back again to problem (A): Why did the universe begin in an unstable low-entropy state?

If we exclude a miraculous act of creation, we have only three scientific answers:

(i) The unstable initial state of the universe is a law of nature.

(ii) This state was produced by a fluctuation.

(iii) The expansion of the universe (coupled to the nuclear reactions in it) produces a decreasing of the (matter-radiation) entropy, so we can explain why there were low-entropy states in some periods of the evolution of the universe.

The first solution is only a way to bypass the problem, while the fluctuation solution is extremely improbable. In fact, the probability of a fluctuation diminishes with the number of particles of the considered system, and the universe is the system with the largest number of particles.

The third solution was sketched by Davies (1994) only as a qualitative explanation. The expansion of the universe is like an external agency (namely: external to the matter–radiation system of the universe) that produces a decreasing of its matter–radiation entropy, not only at $t = 0$, but in a long period of the universe evolution. It is this matter–radiation subsystem within the universe what we have considered as "the universe" up to now, since we have neither considered the gravitational field nor defined an entropy for this field (see Introduction). In this paper we will try to give a quantitative structure to Davies' solution (see also Aquilano and Castagnino, 1996), using an oversimplified cosmological model which yields a first rough numerical coincidence with observational data. Essentially the gravitational field has two different effects that modify entropy in the universe. These two effects can be explained heuristically as follows:

(i) The expansion of the universe behaves like a moving wall creating more space in a room. If the room was full with a very viscous fluid, when the wall moves it creates empty space and therefore a structure where there was nothing. Entropy, in consequence, decreases.

(ii) Gravitational attraction creates condensation of what we can consider "viscous" matter. The motion of these viscous currents is irreversible. Entropy, in consequence, grows.

For simplicity, we will only consider the effect of the first process.

Nevertheless, it is well known that the homogeneous universe expansion alone is a reversible process with constant entropy (Tolman, 1987; Misner *et al.*, 1970). The radiation of the universe is therefore in a thermic equilibrium state ρ_* at any time, and also in thermic equilibrium with almost all the matter. As the radiation is the only important component, from the thermodynamic point of view, we can choose ρ_* as a blackbody radiation state (Peebles, 1993), namely ρ_* will be a diagonal matrix with main diagonal

$$\rho_*(\omega) = ZT^{-3}\frac{1}{e^{\omega/T} - 1} \tag{38}$$

where T is the temperature, ω is the energy, and Z is a normalization constant [Landau and Lifshitz, (1958), equations (60.4) and (60.10)]. The total entropy is

$$S = \frac{16}{3}\sigma V T^3 \tag{39}$$

(Landau and Lifshitz, 1958) equation (60.13)], where σ is the Stefan–Boltzmann constant and V is a comoving volume.

Let us consider an isotropic and homogeneous model of the universe with radius (or scale) a. As $V \sim a^3$, and from the conservation of the energy-momentum tensor and radiation state equation, we know that $T \sim a^{-1}$, and we can verify that S = const. So the irreversible nature of the universe evolution is not produced by the universe expansion alone, even if $\rho_*(t)$ has a slow time variation.

Then the main process that has an irreversible nature is the burning of unstable H in stars (which produces He and, after a chain of nuclear reactions, Fe). This nuclear reaction process has certain mean lifetime $t_{NR} = \gamma^{-1}$ and therefore phenomenologically we can say the state of the universe at time t is

$$\rho(t) = \rho_*(t) + \rho_1 e^{-\gamma t} + O[(\gamma t)^{-1}] \tag{40}$$

where ρ_1 is certain phenomenological coefficient, constant in time, since all the time variation of nuclear reactions is embodied in the exponential law $e^{-\gamma t}$. We can also see on phenomenological grounds that ρ_1 must peak strongly around ω_1, the characteristic energy of the nuclear process.

All these reasonable phenomenological facts can also be explained theoretically:

We know that a real decaying evolution is not perfectly exponential, but has a strong exponential behavior which is modified for short times by the Zeno effect and for long times by the Khalfin effect (Sudarshan *et al.*, 1978). These effects are unimportant for the average times we will consider and they are neglected in (40). With this simplification, (40) can be computed with the theory of Sudarshan *et al.* (1978). It can also be computed with rigged Hilbert space theory of Bohm (1986), Bohm *et al.* (1989), Bohm and Gadella (1989), and Antoniou and Prigogine (1993), as we have already said [cf. (36)]. In Castagnino and Laura (1997a) it is explicitly proved that ρ_1 peaks strongly at the energy ω_1 and becomes a Dirac delta in the limit of large time and small coupling coefficient.

Now we are able to compute the *entropy gap*, namely the (matter–radiation) entropy with respect to the equilibrium state ρ_* [we repeat that we do not consider the eventual entropy of the gravitational field, as in Davies (1994)]. Precisely, if $S = S_{act}$ is the actual entropy of the universe, which will necessarily grow if we take into account all the features of the irreversible evolution of the universe [e.g. the viscous currents of point (ii)] and S_{max} is the maximal possible entropy at a time t, the entropy gap is $\Delta S = S_{act} - S_{max}$. This entropy gap will be the conditional entropy [cf. (37)] of the state $\rho(t)$ with respect to the equilibrium state ρ_* (Mackey, 1989, 1992; Lasota and Mackey, 1985):

$$\Delta S = -\text{tr}[\rho \log(\rho_*^{-1}\rho)] \tag{41}$$

As in (23), and considering only times $t >> t_{NR} = \gamma^{-1}$, we can expand the logarithm to obtain

$$\Delta S \approx -e^{-\gamma t}\, \text{tr}(\rho_*^{-1}\rho_1^2) \tag{42}$$

where we have used (19). Now we can introduce the equilibrium state (38) for $\omega >> T$; then

$$\Delta S \approx -Z^{-1}T^3 e^{-\gamma t}\, \text{tr}(e^{\omega/T}\rho_1^2) \tag{43}$$

where $e^{\omega/T}$ is a diagonal matrix with this function as diagonal. But as ρ_1 is peaked around ω_1, we arrive to a final formula for the entropy gap:

$$\Delta S \approx -CT^3 e^{-\gamma t} e^{\omega_1/T} \tag{44}$$

where C is a positive constant.

6.1. Evolution of the Entropy Gap

As we will see, the expansion of the universe will produce a decreasing of the (matter–radiation) entropy even for large times. So we will focus on computation at times bigger than the decoupling time; therefore $a \sim t^{2/3}$ and $T \sim a^{-1}$. Then we have

$$T = T_0 \left(\frac{t_0}{t} \right)^{2/3} \tag{45}$$

where t_0 is the age of the universe and T_0 the present temperature. Then

$$\Delta S \approx -C_1 e^{-\gamma' t} t^{-2} e^{(\omega_1/T_0)(t_0/t)^{2/3}} \tag{46}$$

where C_1 is a positive constant. Figure 9 is a graphic representation of curve $\Delta S(t)$. It has a maximum at $t = t_{cr_1}$ and a minimum at $t = t_{cr_2}$. We will find an estimate of the physical data before we discuss the curve $\Delta S(t)$. Figure 10 shows ΔS as the difference between S_{max} and S_{act}. Let us compute these critical times. The time derivative of the entropy reads

$$\dot{\Delta S} \approx \left[-\gamma - 2t^{-1} + \frac{2}{3} \frac{\omega_1}{t_0 T_0} \left(\frac{t_0}{t} \right)^{1/3} \right] \Delta S \tag{47}$$

This equation shows two antagonistic effects. The universe expansion effect is embodied in the second and third terms in the square brackets and it acts external to the matter–radiation system such that if we neglect the second term, it tries to increase the entropy gap and therefore it tries to take the system away from equilibrium at any time t (as we will see, the second term is practically negligible). On the other hand, the nuclear reactions embodied

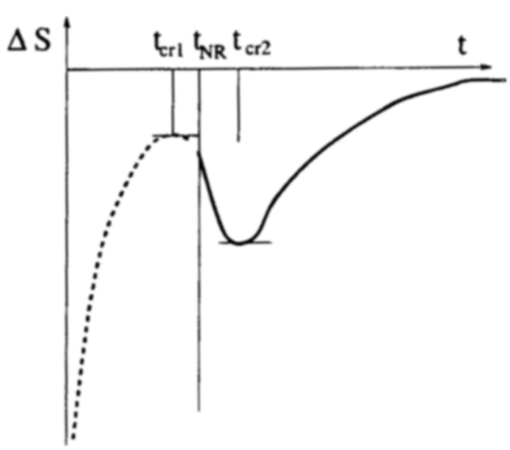

Fig. 9. The evolution of the entropy gap. This figure is only qualitative; scales are not the real ones.

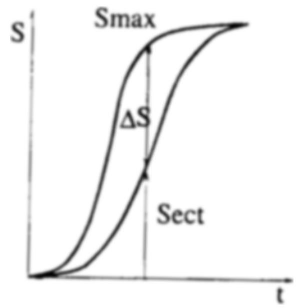

Fig. 10. The evolution of S_{max} and S_{act}. This figure is also only qualitative.

in the γ term try to convey the matter–radiation system toward equilibrium. These effects become equal at the critical time t_{cr} such that

$$\gamma t_0 + 2\frac{t_0}{t_{cr}} = \frac{2}{3}\frac{\omega_1}{T_0}\left(\frac{t_0}{t_{cr}}\right)^{1/3} \tag{48}$$

For almost all reasonable numerical values this equation has two positive roots, such that $t_{cr_1} << t_0 << t_{cr_2}$. Precisely:

(i) For the first root we can neglect the γt_0 term and we obtain

$$t_{cr_1} \approx t_0\left(3\,\frac{T_0}{\omega_1}\right)^{3/2} \tag{49}$$

(this quantity, with minus sign, gives the third, unphysical root).

(ii) For the second root we can neglect the $2(t_0/t_{cr})$ term, and we find

$$t_{cr_2} \approx t_0\left(\frac{2}{3}\frac{\omega_1}{T_0}\frac{t_{NR}}{t_0}\right)^3 \tag{50}$$

where t_{NR} is the mean lifetime of nuclear reaction within the stars.

6.2. Numerical Estimates

We must choose numerical values for the following four parameters:

$\omega_1 = T_{NR}$, the energy or temperature of nuclear reactions.
$t_{NR} = \gamma^{-1}$, the mean lifetime of nuclear reactions.
t_0, the age of the universe.
T_0, the present temperature of the universe.

T_{NR} and t_{NR} can be chosen in the following ranges (Jones and Forman, 1992):

$$T_{NR} = 10^6\text{–}10^8\,\text{K}$$
$$t_{NR} = 10^6\text{–}10^9 \text{ years} \tag{51}$$

while for t_0 and T_0 we can take

$$t_0 = 1.5 \times 10^{10} \text{ years} \quad (52)$$

$$T_0 = 3\text{K}$$

In order to obtain a reasonable result, we choose the lower bounds for T_{NR} and t_{NR} [remember that our model is oversimplified, since we are using only cause (i) to explain the global entropy evolution] and for t_{cr_1} we obtain

$$t_{cr_1} \approx 1.5 \times 10^3 \text{ years} \quad (53)$$

So t_{cr_1} is smaller than the decoupling time and therefore must not be considered, since the physical processes before this time are different than those considered in our model. Also, we must only consider times $t > t_{NR} = \gamma^{-1}$, in order to use (42). So only the r.h.s. of the dashed line of Fig. 9 can be considered.

For t_{cr_2} we obtain

$$t_{cr_2} \leq 10^4 t_0 \quad (54)$$

From (53) and (54) we can see that in fact $t_{\mathrm{cr}_1} << t_0 << t_{\mathrm{cr}_2}$. Thus:

• From t_{NR} to t_{cr_2} the expansion of the universe produces a decrease of entropy, according to Davies' prediction. It also produces a growth of order, and therefore creation of structures like clusters, galaxies, and stars (Reeves, 1993).

• After t_{cr_2} we have a growth of entropy, a decrease of order, and a spreading of the structures: stellar energy is spread in the universe, which ends in a thermic equilibrium (Dicus *et al.*, 1982). In fact, when $t \to \infty$ the entropy vanishes [see (46)] and the universe reaches a thermic equilibrium final state.

$t_{\mathrm{cr}_2} \leq 10^4 t_0$ is the frontier between the two periods. Is the order of magnitude of t_{cr_2} a realistic one? In fact it is, $10^4 t_0 \approx 1.5 \times 10^{14}$ years after the big-bang all the stars will exhaust their fuel (Dicus *et al.*, 1982), so the border between the two periods most likely has this order of magnitude and must also be smaller than this number. This is precisely the result of our calculations. This result is fairly good, since we have only used one of the causes of the decreasing of entropy, namely (i). We cannot use cause (ii) in a homogeneous universe. So in more refined inhomogeneous models it may be that the average values of (51) could be considered. But essentially our oversimplified model proves that low-entropy states within the universe, not only at the beginning, but during a long period of its history, can be reasonably understood.

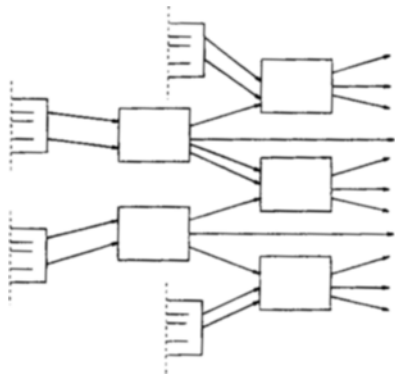

Fig. 11. Bohm diagram of a branch system with continuous creation of instability.

The Bohm diagram of this process is shown in Fig. 11, where we do not have an initial unstable state, as in Fig. 7, but a continuous production of unstable states by an external agency. Figure 11 is as asymmetric as Fig. 7, so it also defines an arrow of time.

In this section we have avoided two major problems:

• The definition of the entropy of the gravitational filed, since we have only worked with the matter–radiation entropy.

• The initial state of the universe, which probably must be described using quantum gravity, because we have only considered time $t > t_{NR}$.

Even though these simplifications allowed us to do the calculation, it is evident that it would be important to have a more complete model to really understand the problem.

7. COORDINATION OF THE ARROWS OF TIME

Let us now go to problem (C). In this section we will follow the spirit of this the paper and base all reasoning on Bohm diagrams, which symbolize specific calculations (like Feynman graphs). Most of these calculations can be found in the literature (the remainder are too elementary, but some of them must be done, so this section cannot be considered complete in all details.).

We can say that the main historical uncorrelated arrows of time are:

(i) BAT. As explained in the Introduction, this arrows goes, at least in our version, from an unstable initial state (the real cause of the process) in any subsystem of the branch system to a stable equilibrium final state of the motion. In a different form this was introduced by Reichenbach (1956). As we will see, it can be considered the *master arrow of time*, since it is the one that coordinates all the others.

(ii) QAT. This goes from the preparation of a quantum state of a scattering experiment to the measurement. In its primitive version, it was the arrow of time related to the collapse of the wave function (a clear time-irreversible quantum process) and was introduced by Bohr and studied in what was called the Bohr–Ludwig program (Ludwig, 1979).

(iii) GAT. Usually it is postulated that space-time is a time-oriented Riemannian manifold (Lichnerowicz, 1964) that defines two different classes of light semicones, the future and the past ones, all over the manifold. One orientation is conventionally called the past and the other orientation the future.

Let us see how these three arrows are related.

In its final version QAT considers that the "in" preparation states are related to the notion of *states of the system* and the "out" measured states are related to the notion of *observable* (Bohm, 1995; Bohm *et al.*, 1995b; Antoniou *et al.*, 1995). Since the notions of state and observable are substantially different, we could conclude that QAT is substantially defined, at least experimentally. Nevertheless, both states and observables are mathematically symbolized by matrices, so one may argue that on theoretical grounds the above difference is only conventional. Somehow it seems that the difference is really only contained in the mind of the physicist making the experiment, who knows which are the matrices that correspond to the prepared states and which are the matrices that correspond to the measured states. In fact, it is not evident that the difference between preparation and measurement would be substantial in a natural spontaneous scattering process. But if we consider the scattering process not as an isolated phenomenon, but within the branch system of the universe, we find that it has the diagram inside the dotted box of Fig. 7, namely Fig. 12, where we can see the source of energy that accelerates the incident particles (because there must always be a source related to the acceleration process) is necessarily produced by a previous unstable state (the half box on the left of Fig. 12). Figure 12 is then asymmetric and we see in this perspective that the difference is really substantial, and it defines a local QAT that coincides with the global BAT, as is evident from Fig. 7. So QAT = BAT.

It is quite evident that locally (i.e., in the laboratory) GAT is oriented in such a way that it coincides with BAT. As far as we know, this also

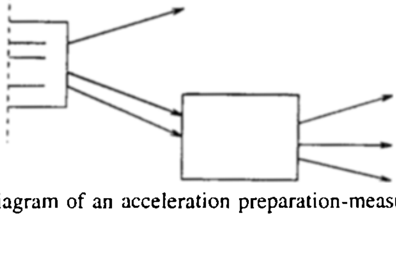

Fig. 12. Bohm diagram of an acceleration preparation-measurement process.

happens in all the observable universe, and we can conclude that globally GAT = BAT, and as QAT = BAT, all three arrows coincide.

But we can try to establish the relation GAT ~ QAT directly. In fact, the geometry of space-time is equivalent to a gravitational (time-variable in general) field that produces (or destroys) particles. The S-matrix of this process generally has an infinite set of poles or resonances (Castagnino *et al.*, 1995a; Castagnino and Lombardo, 1996) and therefore we can find all the elements of a usual scattering process in it, namely the ϕ_- and ϕ_+ spaces, growing and decaying states, etc. The only difference is that now we are dealing with an external agency, the gravitational field, that produces the scattering process, and it is a time-dependent field, so the corresponding diagram is not the one of Fig. 7, but the one of Fig. 11. But as both diagrams are time-asymmetric, we have an arrow of time defined in both diagrams. Thus Fig. 11 also symbolizes a curved space-time particle creation (or annihilation) process where the external agency is the variable gravitational field, participating in the global branch system, since its variation is a consequence of the unstable initial conditions of the universe (so the semi-boxes of Fig. 11 are well drawn). Therefore the arrow of time defined from the asymmetry of Fig. 10 is the GAT, since the two orientations or classes of semicones can be named the past class, oriented toward the cause of the process (i.e., the "big bang") and the future class, oriented in the opposite direction. So GAT = QAT, also in a direct way.

Now we can see the relation with the other arrows of time:

The quantum Gel'fand triplet structure is completely equivalent to the classical Gel'fand triplet structure and they can be related to a Wigner integral (Antoniou and Tasaki, 1991, 1993) But the latter structure allows us to define classical Lyapunov variables (Castagnino *et al.*, 1997c). Therefore QAT = TAT. But we can directly go from QAT to TAT, as can be seen if we consider the quantum definition of entropy (37) and rephrase what we have said in the classical case.

EMAT is just GAT, because it is the choice of the past semicone to define the retarded solutions as the causal ones, so EMAT = GAT.

Let us now go to PAT. Of course, to understand this arrow completely we would need to use biological and psychological notions, which are beyond our field of research. Nevertheless, PAT is the intuitive and oldest notion we have about the flow of time, so we cannot leave the PAT completely outside of our research. Therefore we will make some observations to convince the reader that our formalism introduces enough elements to allow a future convincing proof that PAT = BAT.

Let us first observe that our brain, considered as a machine, is also included in the branch global system of the universe. In fact, neurons burn carbohydrates, going from unstable to stables equilibrium states. Therefore

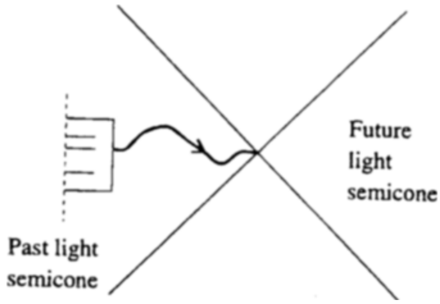

Fig. 13. Bohm diagram of a message in space-time.

it is not surprising that we can feel this physical process as the psychological flow of time.

There are two other features that are essential to the PAT that can be treated with our formalism, namely, on one hand, the "historical" nature of PAT, in the sense that we can conceive time as an increase of history that goes from a known past to an unknown future (Schrödinger, 1950), and on the other hand, the feeling that we are all in a common "present", in the sense that we share a common history. Let us first consider that every message is equivalent to the transport of a certain amount of energy. This energy must be necessarily created by a decaying process near the emission point. Then let us draw what we can consider the Bohm diagram of the path of a message in space-time (Fig. 13). It is a didactic superposition of a past light semicone with its vertex at the reception point and the Bohm diagram of a decaying process, showing the source, which must necessarily be placed inside the past light cone. Now we can draw the space-time path of an observer receiving all kinds of messages from the universe (Fig. 14). The amount of information he has (his history) grows when the observer goes from the left (past) to the right (future). As the growth of history is considered by the observer as his PAT, this arrow coincides with BAT (and GAT). Now let us consider two near observers (Fig. 15). They share a common portion of history (the shaded

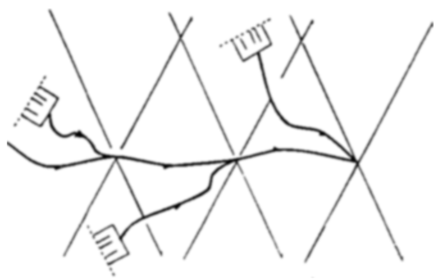

Fig. 14. Bohm diagram of the increasing of an observer history.

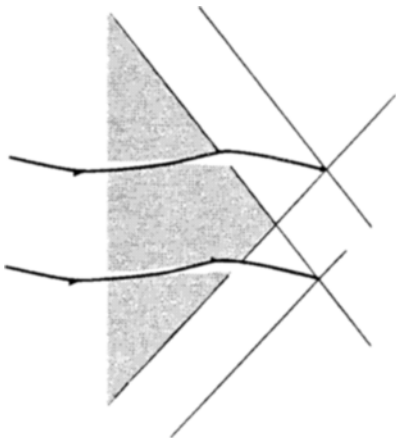

Fig. 15. Bohm diagram of the common notion of the present of two observers.

area of Fig. 15). Let us now go to the classical limit where light cones angles are π. For simplicity we consider that the two observers do not move with respect to the reference system, so their paths are parallel straight lines, normal to the light cones (see Fig. 16). This is of course the sensible psychological case. The observers will say that they are in a common (universal, absolute) present if their histories, which they both know, are exactly the same. In this case the notion of the present is an absolute one. Of course in the general relativistic case this absolute notion is lost (Fig. 15), but this is not the psychological case, since we do not perceive any relativistic correction. The absolute present that we feel is, of course, a classical notion. This is all that we can say about PAT. Most likely PAT is the combined feeling of our brain burning carbohydrates, the growth of history, and the fact that this history is shared by other observers when they are in our present. Then also PAT = BAT.

Finally, let us consider CAT. The relation of this arrow to the others is model dependent. If we choose an expanding model of the universe, it coincides with all the others. If we choose a model of a expanding–contracting universe, it coincides with all others only in the expansion period. These facts can be immediately seen by comparing CAT with GAT in these models. The only thing we can add is that GAT does not change when we go from the expansion to the contraction period in the latter model, since this model is an oriented manifold, with a GAT that cannot change, and is fixed by the BAT, defined by the initial unstable state, which is always in the past of every event of the manifold (about the relation of QAT and GAT in this case let us observe that a contracting universe produces particles like an expanding one). So CAT is not a good candidate for a master arrow of time. In our

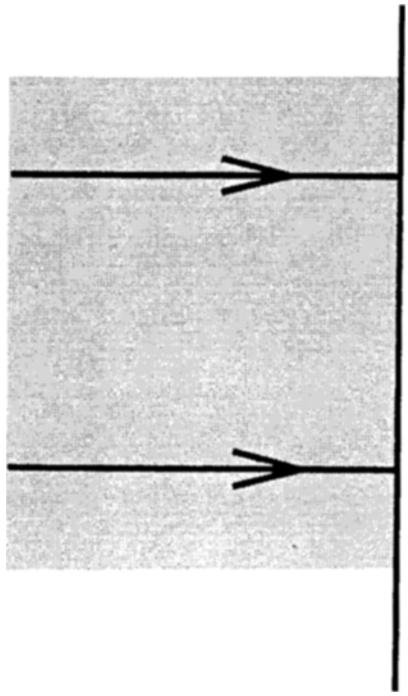

Fig. 16. Figure 15 in the classical case.

discussion this role is played by BAT and also by GAT, which are much easier to relate to the other arrows than CAT.

So, with the exception of CAT, which is model dependent, all the arrows are coordinated.

8. CONCLUSION

Even if much work must be done to complete the details, the main lines of the landscape are already finished. Time asymmetry can be considered as a global symmetry breaking, if we use the Gel'fand triplet structure, in the most convenient and economical way. Let us comment on this statement:

Global: The arrow of time is clearly global. If it were a local phenomenon, we could construct different arrows of time in different laboratories, which obviously is not the case. Nevertheless some local laboratory physicists might find this concept difficult to understand. They would ask, e.g., what would happen if suddenly a Maxwell demon changed the direction of the velocities of every point: would the arrow of time change? This proposition has only (a local) sense if the demon changes all the velocities *inside* the laboratory and leaves the velocities *outside* the laboratory untouched. Since there is a (global) arrow of time external to the laboratory, we have an extra arrow of time to compare with and verify that there is a change on the direction of velocities inside the laboratory. Then the answer will be that there is no change in the arrow of time, since it is defined by the motion exterior to the laboratory. We will only see that things inside the laboratory move in an opposite direction. Then, as the arrow of time is global, we may ask ourselves what would happen if *the demon changes the velocities of all the points of the universe?* Then we can say that this task (a) is practically impossible, and (b) is also theoretically impossible, because if the demon changes all the velocities (when we are sleeping) we would not see any change (when we wake up), since we will find that all arrows of time pointing again in the same common direction (even if different than the previous one) and we have no extra arrow of time to compare. So there is no real change and we can say that the only thing that the global demon has accomplished is to pass from space Φ_- to space Φ_+, which, being identical, have only a conventional difference.

So the task of the global demon is not only practically impossible, but theoretically meaningless. For this reason restricted dynamics theory confines the dynamics to space Φ_- and forbids space Φ_+, precisely because the *global* inversion T: $\Phi_- \to \Phi_+$ is practically impossible and theoretically meaningless

Gel'fand triplet structure: It is essential to see the generalized spontaneous symmetry breaking. If we remain within the old structures, like classical Liouville–Hilbert space, we do not see it. For example, all initial conditions belong to this space and have two equilibrium states, one at the far past and one at the far future, for mixing evolutions. Thus, these states cannot break the time symmetry.

Convenient: Using the new spaces of the triplet, we can expand the states in decaying or growing bases of eigenstates of the Hamiltonian and compute their time evolution easily. These eigenstates do not exist in usual Hilbert or Liouville–Hilbert spaces.

Economical: To introduce irreversibility we must add something (e.g., coarse-graining) or make some changes. We believe that the minimal change is just $\mathcal{S} \to \phi_-$.

We hope that all these reasons persuade the reader that we are on the right road. But we are also sure that the polemics will continue.

ACKNOWLEDGMENT

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